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TABLES OF THE HYPERGEOMETRIC DISTRIBUTION FUNCTIONS

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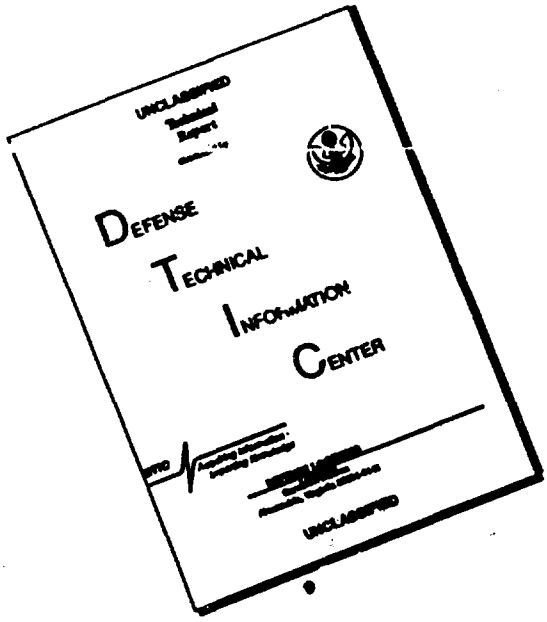

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FOREWORD

Tables and graphs based on the hypergeometric distribution are presented for use in determining the confidence interval of the sample estimate of the number of defectives in a finite population. Similarly, the sample size can be determined which would give a certain quality level as the lower bound for a selected confidence level. The hypergeometric distribution is particularly suited for small populations (less than 1,000) where a saving in the sample size is desired even at the expense of some loss in precision of the estimate.

The tables of point and cumulative probabilities are tabulations of selected sample and population combinations. The selected sample sizes range from 4 to 40 and the population, from 50 to 1,000.

For those that have access to an IBM 1401 Model B-4, 8K memory, the computer program is included as Appendix D.

The authors wish to express their appreciation to A. Ohta and K. Thornton for editing and assembly of the tables and to J. Mitchell for supervising the computer tabulation.

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INTRODUCTION

In quality evaluation, the basic question is: "What is the quality level of the stockpile in question?" More often this question is put in the following form: "How large must the sample be to give a certain level of assurance that the stockpile is no worse than X% defective if no defectives are observed in the sample?" In this latter form, the requirement is not for a precise estimate of the stockpile quality level but rather some assurance that the quality is not below a specified level. In this situation, the implication is that there is some willingness to sacrifice some precision if a reduction in the sample size required can be realized.

Most approaches until recently have been based on the binomial distribution. But in cases where the stockpile is small, say less than 500, and the unit item cost high, the binomial has not been a very satisfactory model. As is normally the case in quality evaluation where the populations are small and sampling is without replacement, it appeared that the hypergeometric distribution was the more realistic model to use, but until the advent of the modern-day computer, the formidable task of calculating the probabilities on a desk calculator prevented its use.

Of primary concern to the Oahu Laboratory is coping with small stockpiles of high unit cost weapons. What is desired is a method whereby stockpiles of extremely high or low quality (percent operability) can be readily detected using a minimal size sample. For stockpiles falling in between, additional samples must be tested if greater precision in the quality estimates is desired. As a result, a study was made of a two-stage sampling method based on the hypergeometric distribution and using 95% operability at the 90% confidence level as the lower bound for "good" stockpiles.

APPROACH TO THE PROBLEM

Based on the hypergeometric distribution two main mathematical approaches are proposed.

Approach I

The first approach may be stated in this mathematical form

$$(1) \qquad P(D|S,N,M) = \frac{\binom{M}{D}\binom{N-M}{S-D}}{\binom{N}{S}} = \frac{\left[\frac{M!}{(M-D)!\ D!}\right]\left[\frac{(N-M)!}{(N-M-S+D)!\ (S-D)!}\right]}{\frac{N!}{(N-S)!\ S!}}$$

where: S = Sample size
N = Population size
D = Sample defectives
M = Population defectives

and obviously

S -D = Sample Non-defectives
N-M = Population Non-defectives

Equation (1) states that the probability of obtaining D defectives in a sample of size S, given M defectives in a population of N items, is equal to the number of ways of drawing D out of M items times the number of ways of drawing S-D out of N-M items divided by the number of ways of drawing S out of N items. In stockpile quality estimation it is desired to find the confidence interval for the number of defectives in a finite population. Since M is not known, an upper bound on the true M is sought. Call this bound M_u. First assume that $M_u = M_1$. Then look up in an appropriate table (reference (4)) the sum of the probabilities of drawing D or less defectives in the sample. If this sum is less than α, the significance level (e.g. .10), the proper M_u should be less than M_1. Then choose $M_u = M_2$, $M_2 < M_1$ and repeat the above procedure until

$$\sum_{D_i=0}^{D} P(D_i|S,N,M_u-1) \leq \alpha \leq \sum_{D_i=0}^{D} P(D_i|S,N,M_u)$$

Then select M_u-1 or M_u as the upper bound depending on which corresponding sum is closer to α.

The maximum likelihood value is given as $\hat{M} \leq \frac{D}{S}(N+1)$

(reference 1, p. 294) or more completely $\frac{D}{S}(N+1) - 1 \leq \hat{M} \leq \frac{D}{S}(N+1)$

(reference 3, p. 3). References for this approach are found in (1), (2), and (3).

Approach II

The approach followed is to assume that populations with M defectives where M ranges from D to (N-S+D) are tested randomly. In this case $P(D|S,N,M_i)$ is related to $P(D|S,N,M_D)$, $P(D|S,N,M_{D+1})$, ... $P(D|S,N,M_{N-S+D})$. The probability of the observed sample coming from a population with M_i defectives is given as:

$$(2) \qquad P(M_i \mid D,S,N) = \frac{P(D|S,N,M_i)}{\sum_{M=D}^{N-S+D} P(D|S,N,M)} \qquad \text{where, } D \leq M_i \leq N-S+D$$

Equation (2) states that if, from a population of size N, a sample S is drawn and D defectives are observed, the probability of M_i defectives in the population is equal to the ratio of the probability of that set of M_i, N, S, and D to the total sets of N, M_j, S, D where M_j is allowed to range from D to (N-S+D). The number of defectives in the population cannot be less than the number of defectives observed in the sample nor greater than the difference between the total population N, and (S-D) (sample non-defectives).

The denominator $\sum_{M_j=D}^{N-S+D} P(D|S,N,M_j)$ can be shown equal to $\frac{N+1}{S+1}$, a constant. This makes it valid to tabulate P(D S, N, M) instead of P(M S, N, D) for checking purposes. In the form of equation (1), equation (2) becomes the derived equation:

$$(3) \qquad P(M \mid D,S,N) = \frac{\binom{M}{D}\binom{N-M}{S-D}}{\binom{N+1}{S+1}} = \frac{S+1}{N+1} P(D|S,N,M)$$

For brevity, let $P(M_i \mid D,S,N) = P(M)$. Then these recurrence relationships are very useful for computational purposes:

$$(4) \quad \frac{P(M)}{P(M+1)} = \frac{(M-D+1)\ (N-M)}{(M+1)\ (N-M-S+D)}$$

$$(5) \quad \frac{P(M)}{P(M-1)} = \frac{(M)\ (N-M-S+D+1)}{(M-D)\ (N-M+1)}$$

From (4) and (5) it can be seen that $P(M) > P(M+1)$, where they exist, as long as $(M-D+1)\ (N-M) > (M+1)\ (N-M-S+D)$ and similarly $P(M) > P(M-1)$ as long as $(M)\ (N-M-S+D+1) > (M-D)\ (N-M+1)$. It follows that the maximum likelihood integer $\hat{M}$ may be expressed as:

$$(6) \quad \frac{D}{S}(N+1) - 1 \leq \hat{M} \leq \frac{D}{S}(N+1)$$

There will be two $\hat{M}$ values where the extreme right and left expressions are: (a) integers and (b) exist. Since equations (4), (5) and (6) show that the probabilities decrease from the maximum likelihood value, serial computations should start with $\hat{M}$ as shown below in equations (7a) and (7b). (Note: $\hat{M}$ differs from that in [Reference 1] and [Reference 3].)

$$(7a) \quad P(\hat{M}) \geq P(\hat{M}-1) \geq P(\hat{M}-2) \geq \ldots . \ P(\hat{M}-k)$$

$$(7b) \quad P(\hat{M}) \geq P(\hat{M}+1) \geq P(\hat{M}+2) \geq \ldots . \ P(\hat{M}+i)$$

$$(7c) \quad \sum_{R}^{T} P(M_j) = 1 - \alpha$$

If $P(\hat{M}+i) \geq P(\hat{M}-k)$, $P(\hat{M}+i)$ is added to $\sum P(M_j)$ which starts with $P(\hat{M})$; otherwise $P(\hat{M}-k)$ is added. Equation (7c) states that R to T

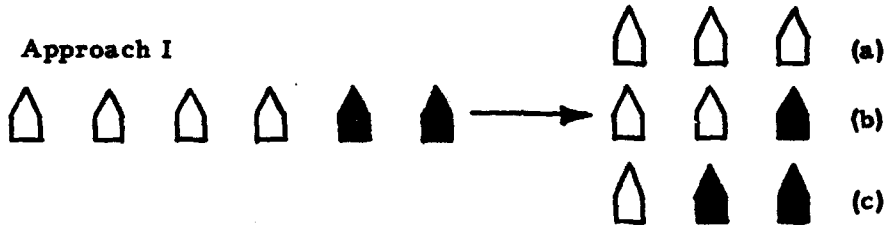
is the range of M defectives in the population when the sum of all the higher probabilities is equal to $1-\alpha$, the confidence level.

$$(8) \qquad \sum_{D}^{L} P(M_j) = 1-\alpha ,$$

L being determined by computations. Equation (8) gives the upper bound of M_j or the so-called one-tail test for small values of D. For D = 0 and D = S, the one-tail and two-tail tests coincide. For most values of D, the probabilities form an asymmetric as well as a discrete distribution. The asymmetries are illustrated in figure 1 where curved lines are drawn through point probabilities for N = 50, S = 8, and D = 0 to 4.

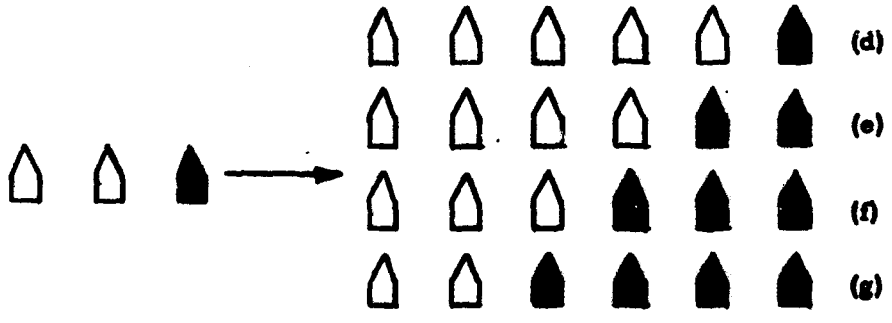
Approaches I and II may perhaps be better illustrated by the use of black and white missiles.

Approach I



There are six missiles in the population - four white and two black. The three possible outcomes in a sample of three are labeled (a), (b), and (c); the chance or probability of drawing any one is determined by equation (1).

Approach II. In stockpile quality evaluation it is desirable to take approach II which is the more realistic statistical model. In this case the population is estimated from a known sample.



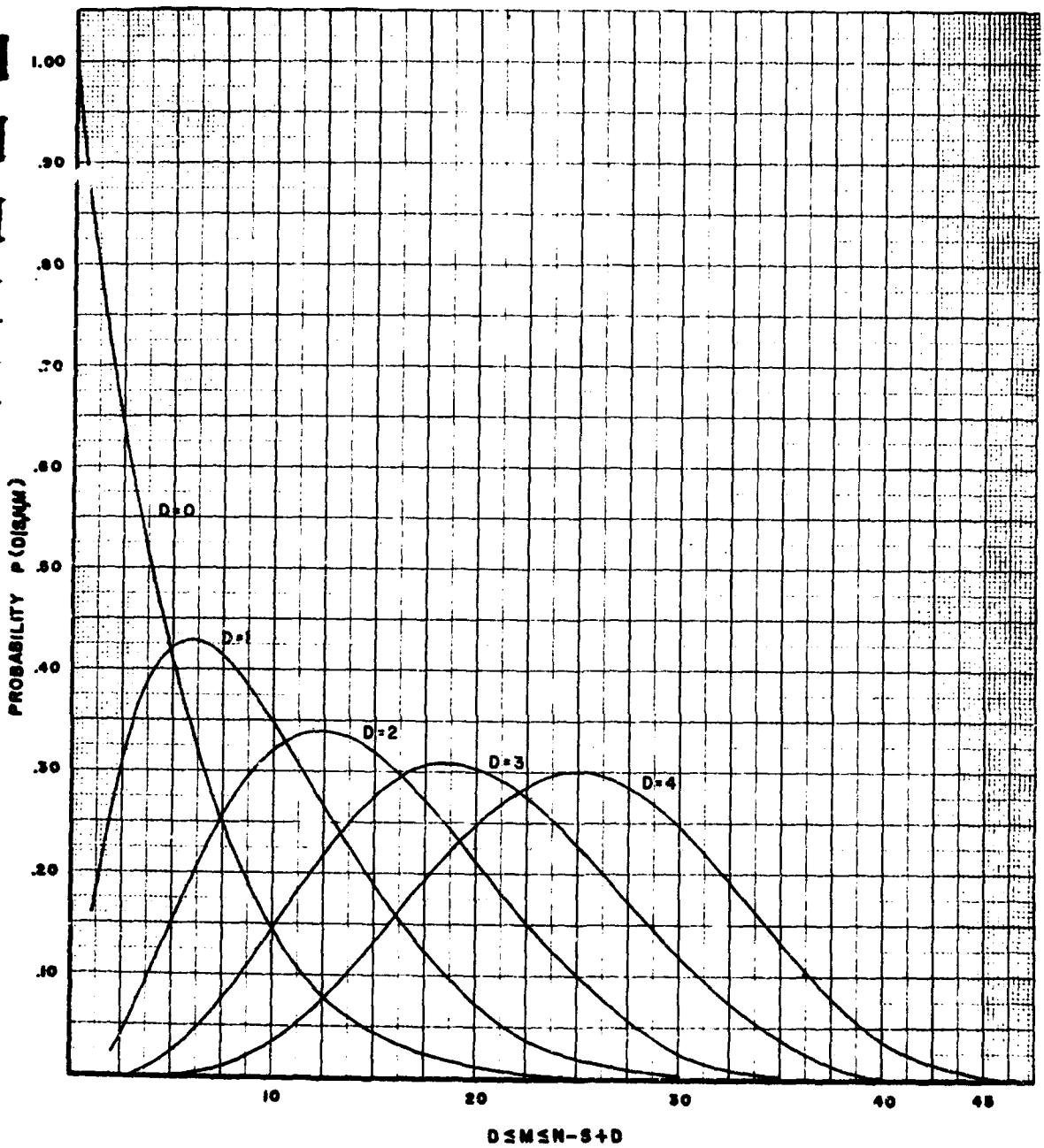


FIGURE 1. CHANGE IN SHAPE OF GENERALIZED POINT PROBABILITY CURVES DUE TO THE NUMBER OF DEFECTIVES PRESENT IN A SAMPLE.

Here, from a population of six, is a sample of three wherein two white and one black missiles were noted. The four possible populations from which this sample could have been drawn are: (d), (e), (f), and (g). The chances or probability of any one of the populations being the one from which this particular sample is drawn is determined by equation (3).

Symmetries

Since $\frac{S+1}{N+1}\ P(D|S,N,M) = P(M|S,N,D)$ by equation (3), symmetries given by Lieberman and Owen (Reference 4) for $P(D|S,N,M)$ apply in many instances where S and N are fixed.

(9) $$P(M = M_1 | D_1, S, N) = P(M = N-M_1 | S, S-D_1, N)$$

(10) $$\sum_{M=A}^{B} P(M|S, D = D_1, N) = \sum_{M=N-A}^{N-B} P(M|S, D = S-D_1, N)$$

Equations (9) and (10) show that tables need only involve half the sample size.

Population defectives M given and sample size S unknown

(11) $$P(M|D,S,N) = P(S|D,M,N)$$

(12) $$\sum_{S=D}^{N-M+D} P(S|D,M,N) = \frac{N+1}{M+1}$$

Equations (11) and (12) give the basic equations for the problem of sample size estimation when N, D, and M are known. Any table for M defectives may be used by interchanging S for M.

Two Stage Sampling

Out of a total allowable sample of size S from a population N, an initial sample S_1 is tested and D_1 defectives are found. By means of a

table similar to Appendix C, it is found that x defectives gives the desired percent operability and y defectives (y>x) do not. If $D_1 \leq x$ then the lot is accepted. If $D_1 \geq y$, the lot is rejected. If $x < D_1 < y$, then the remainder of S, called S_2, is sequentially tested. If at any time a total of $D_1 + C$ defectives are found, the lot is rejected, since the total sample S would have at least $D_1 + C$ defectives. If the total sample S is tested and $(D_1 + C - 1)$ defectives are found, the lot is accepted. $D_1 + C$ is determined from the probability table for N, S, and M, the number of defectives that will be tolerated.

Best Sample Size S for a Given D

For some values of D sample defectives, a large sample size S is required to reach the desired percent operability for a given confidence level. In these cases, the percent operability should perhaps be lowered to the point where the additional sample units give less than some pre-selected gain value in percent operability. This is illustrated in figure 2, which is patterned after the graphs in Appendix B.

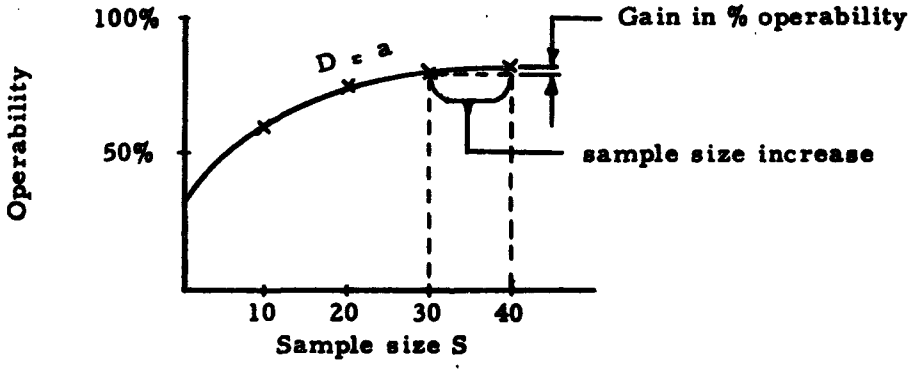


FIGURE 2. SAMPLE SIZE VERSUS OPERABILITY.

Here for D = a, an additional sample of 10, from 30 to 40 will result in a very small gain in percent operability and therefore, S = 30 may be the better choice of sample size.

RESULTS

Tables and Graphs

The table and graphs in Appendices A and B are useful as "quick look" references. The table gives the range for population defectives for certain fixed confidence levels. The graphs give different population lines in terms of percent operability versus fixed sample sizes. Smooth curves are drawn through interpolated points. Only the lower percent operability values are plotted for clarity.

Print-out of Probabilities

A sample of the computer print-out of probabilities is given in Appendix C. Point probabilities and sums for $P(D|S, N, M)$ were tabulated instead of $P(M|D, S, N)$ for purposes of checking with other tables.

The abbreviations are:

S = Sample size
N = Population size
D = Sample defective
M = Population defective
P = $P(D|S, N, M)$

$$\text{SUM} = \sum_{M=R}^{T} P(D|S, N, M)$$ where the sum contains the highest probabilities from R to T

$$\text{CONF} = \frac{\sum_{R}^{T} P(D|S, N, M)}{\sum_{M=D}^{N-S+D} P(D|S, N, M)}$$

$$\text{INTERVAL} = \begin{cases} \text{Lower M} \\ \dfrac{\text{Lower M}}{N} \\ \text{Upper M} \\ \dfrac{\text{Upper M}}{N} \end{cases}$$

LEFT SUM = Sum of decreasing probabilities to the left of maximum likelihood

RIGHT SUM = Sum of decreasing probabilities to the right of maximum likelihood

A one-tail test is possible from this type of table. Assume $1-\alpha$ is the confidence level, $Q = \sum_{M=D}^{N-S+D} P(D|S, N, M)$, and

$$R = \sum_{M=\hat{M}+1}^{N-S+D} P(D|S, N, M).$$

Compute αQ and subtract from R. Then trace back in the "Right sum" column until a value just exceeding $R - \alpha Q$ is found. The value of upper M in the same row is the one-tail upper bound on the population defectives.

Program for the Hypergeometric Series

The symbolic language program developed for the IBM 1401 computer, Mod B4, is given in Appendix D. Accuracy in computation of factorials was mainly accomplished by having the decision to multiply by a number from the numerator or divide by a number from the denominator depend upon the number of leading zeros resulting from the previous calculation. Individual probabilities were then usually accurate to ten places and sums of probabilities to eight.

The abbreviations used in the print-out in Appendix D are:

PG LIN	= Page and line identification
CT	= Count for instruction or reserved storage
OP	= Operation instruction
A OPERAND	= A or I address of instruction
B OPERAND	= B address of instruction
D	= D character modification of the basic instruction

REFERENCES

[1] Deming, William Edwards, Some Theory of Sampling, New York, John Wiley and Sons, 1950, p 294

[2] Katz, Leo, Confidence Intervals for the Number Showing a Certain Characteristic in a Population When Sampling is Without Replacement Journal of the American Statistical Association v. 48 (1953), 256-261

[3] Odell, Patrick L., Tables and Graphs for Determining an Upper Confidence Bound on the Number of Defectives in a Finite Population (U)

U. S. Naval Nuclear Ordnance Evaluation Unit, Albuquerque, New Mexico, 1960, p. 1

[4] Lieberman, G. J. and Owen, D. B., Tables of the Hypergeometric Probability Distribution, Stanford University Press, 1961.

APPENDIX A: TABLES OF POPULATION DEFECTIVES FOR CERTAIN CONFIDENCE LEVELS.

N	S	D	95% Confidence Levels		99% Confidence Levels	
			M_L	M_U	M_L	M_U
20	8	0	0	10	0	13
		1	1	17	1	20
		2	4	26	4	27
		3	9	31	7	33
		4	14	37	12	36
	10	0	0	8	0	10
		1	1	14	1	17
		2	3	20	3	23
		3	7	26	6	28
		4	11	31	9	32
	12	0	0	7	0	9
		1	1	12	1	14
		2	3	17	2	19
		3	6	22	5	24
		4	9	26	8	28
40	8	0	0	12	0	16
		1	1	21	1	24
		2	3	28	4	33
		3	10	37	9	40
		4	16	44	14	44
	10	0	0	10	0	13
		1	1	17	1	20
		2	4	23	3	27
		3	8	31	7	34
		4	12	37	11	40
	12	0	0	9	0	11
		1	1	15	1	17
		2	3	21	3	24
		3	7	27	6	29
		4	10	32	9	34
60	8	0	0	11	0	14
		1	1	20	1	24
		2	4	26	3	30
		3	12	46	10	47
		4	18	51	16	54
	10	0	0	9	0	12
		1	1	16	1	20
		2	5	25	4	28
		3	9	34	8	40
		4	14	43	12	44
	12	0	0	10	0	12
		1	1	17	1	20
		2	4	23	3	28
		3	7	31	6	35
		4	12	38	10	40
80	8	0	0	17	0	21
		1	2	25	1	33
		2	7	40	5	44
		3	13	55	11	56
		4	21	69	18	68
	10	0	0	14	0	18
		1	1	24	1	28
		2	5	33	4	37
		3	10	42	8	46
		4	16	50	14	53
	12	0	0	12	0	15
		1	1	20	1	24
		2	4	28	3	32
		3	8	36	7	39
		4	13	43	11	46
90	8	0	0	19	0	24
		1	2	27	1	32
		2	7	36	6	39
		3	15	47	12	48
		4	24	57	20	70
	10	0	0	16	0	20
		1	1	27	1	31
		2	6	38	5	42
		3	11	49	9	51
		4	18	58	16	60
	12	0	0	13	0	17
		1	1	23	1	27
		2	5	32	4	36
		3	9	40	8	45
		4	15	49	13	51

N	S	D	95% Confidence Levels		99% Confidence Levels	
			M_L	M_U	M_L	M_U
100	8	0	0	21	0	27
		1	2	34	1	42
		2	8	48	6	55
		3	17	61	15	67
		4	26	74	23	78
	10	0	0	18	0	22
		1	2	31	1	36
		2	6	42	5	47
		3	13	53	10	57
		4	20	63	17	67
	12	0	0	15	0	19
		1	1	25	1	30
		2	5	36	4	40
		3	10	46	9	50
		4	16	56	14	58
120	8	0	0	26	0	33
		1	2	40	1	48
		2	10	54	7	62
		3	19	70	16	81
		4	31	85	27	91
	10	0	0	21	0	27
		1	2	30	1	36
		2	7	48	6	54
		3	15	64	12	69
		4	24	76	20	83
	12	0	0	18	0	23
		1	1	31	1	36
		2	6	43	5	48
		3	12	55	10	60
		4	19	66	16	70
	15	0	0	15	0	19
		1	1	25	1	30
		2	6	36	4	41
		3	10	46	8	50
		4	15	54	12	59
	17	0	0	13	0	17
		1	1	22	1	27
		2	4	32	3	36
		3	8	40	7	45
		4	13	48	11	52
140	8	0	0	30	0	38
		1	3	52	2	60
		2	11	71	8	78
		3	22	90	19	95
		4	34	104	31	109
	10	0	0	25	0	32
		1	2	43	1	50
		2	8	59	6	66
		3	17	74	14	81
		4	27	88	23	94
	12	0	0	21	0	27
		1	2	37	1	43
		2	7	51	5	57
		3	14	64	11	70
		4	22	76	19	82
	15	0	0	17	0	22
		1	1	30	1	35
		2	6	42	4	47
		3	11	53	9	58
		4	17	63	15	68
	17	0	0	15	0	19
		1	1	26	1	31
		2	5	37	4	41
		3	10	48	8	52
		4	15	58	13	63
160	8	0	0	35	0	44
		1	3	60	2	69
		2	12	82	9	89
		3	26	101	22	106
		4	41	119	36	122
	10	0	0	29	0	37
		1	2	49	1	57
		2	9	68	7	76
		3	20	86	16	92
		4	31	101	27	106

N	S	D	95% Confidence Levels		99% Confidence Levels	
			M_L	M_U	M_L	M_U
160	12	0	0	25	0	31
		1	2	45	1	49
		2	8	59	6	64
		3	16	74	13	81
		4	25	87	21	94
	15	0	0	20	0	25
		1	2	35	1	40
		2	6	46	5	55
		3	13	61	10	67
		4	20	73	17	79
	17	0	0	18	0	23
		1	1	28	1	34
		2	5	43	4	48
		3	11	55	9	60
		4	17	65	15	71
180	8	0	0	39	0	48
		1	3	66	2	77
		2	14	88	11	101
		3	28	110	25	122
		4	47	134	39	140
	10	0	0	33	0	41
		1	2	55	1	64
		2	11	77	9	86
		3	22	96	18	104
		4	35	114	30	122
	12	0	0	28	0	35
		1	2	44	1	50
		2	7	58	6	66
		3	15	73	14	80
		4	25	87	24	94
	15	0	0	23	0	29
		1	2	39	1	45
		2	7	54	6	61
		3	16	68	11	75
		4	25	81	19	87
	17	0	0	21	0	26
		1	2	36	1	41
		2	6	49	5	55
		3	12	61	10	67
		4	19	73	16	80
200	8	0	0	44	0	54
		1	3	74	2	84
		2	16	102	11	112
		3	32	127	28	132
		4	51	148	43	154
	10	0	0	37	0	46
		1	2	62	1	72
		2	11	86	9	96
		3	26	107	22	116
		4	39	127	35	136
	12	0	0	31	0	40
		1	2	50	1	58
		2	10	68	7	77
		3	20	85	17	93
		4	31	100	26	108
	15	0	0	26	0	33
		1	2	44	1	52
		2	8	61	6	69
		3	15	76	13	84
		4	24	91	21	98
	17	0	0	24	0	30
		1	2	40	1	47
		2	7	54	5	62
		3	14	67	12	75
		4	22	80	20	87
	20	0	0	19	0	25
		1	1	34	1	40
		2	6	48	5	54
		3	12	60	9	67
		4	18	71	16	77

APPENDIX A - CONTINUED

N	S	D	95% Confidence Levels		99% Confidence Levels	
			M_L	M_U	M_L	M_U
250	8	0	0	26	0	28
		1	4	34	2	36
		2	10	42	8	44
		3	17	49	15	51
		4	24	56	22	58
	10	0	0	30	0	32
		1	3	39	2	41
		2	9	47	7	49
		3	15	55	13	57
		4	21	63	19	65
	12	0	0	34	0	36
		1	3	43	2	45
		2	8	52	6	54
		3	13	60	11	62
		4	19	68	17	70
	15	0	0	39	0	41
		1	3	48	2	50
		2	7	57	5	59
		3	12	65	10	67
		4	17	74	15	76
	17	0	0	43	0	45
		1	2	53	1	55
		2	7	62	5	64
		3	11	71	9	73
		4	16	79	14	81
	20	0	0	48	0	50
		1	2	58	1	60
		2	6	67	4	69
		3	10	76	8	78
		4	14	85	12	87
	25	0	0	57	0	59
		1	2	67	1	69
		2	5	77	3	79
		3	8	86	6	88
		4	12	95	10	97
300	8	0	0	26	0	28
		1	4	34	2	36
		2	10	42	8	44
		3	17	49	15	51
		4	24	56	22	58
	10	0	0	30	0	32
		1	3	39	2	41
		2	9	47	7	49
		3	15	55	13	57
		4	21	63	19	65
	12	0	0	34	0	36
		1	3	43	2	45
		2	8	52	6	54
		3	13	60	11	62
		4	19	68	17	70
	15	0	0	39	0	41
		1	3	48	2	50
		2	7	57	5	59
		3	12	65	10	67
		4	17	74	15	76
	17	0	0	43	0	45
		1	2	53	1	55
		2	7	62	5	64
		3	11	71	9	73
		4	16	79	14	81
	20	0	0	48	0	50
		1	2	58	1	60
		2	6	67	4	69
		3	10	76	8	78
		4	14	85	12	87
	25	0	0	57	0	59
		1	2	67	1	69
		2	5	77	3	79
		3	8	86	6	88
		4	12	95	10	97
350	8	0	0	26	0	28
		1	4	34	2	36
		2	10	42	8	44
		3	17	49	15	51
		4	24	56	22	58
	10	0	0	30	0	32
		1	3	39	2	41
		2	9	47	7	49
		3	15	55	13	57
		4	21	63	19	65
	12	0	0	34	0	36
		1	3	43	2	45
		2	8	52	6	54
		3	13	60	11	62
		4	19	68	17	70
	15	0	0	39	0	41
		1	3	48	2	50
		2	7	57	5	59
		3	12	65	10	67
		4	17	74	15	76
	17	0	0	43	0	45
		1	2	53	1	55
		2	7	62	5	64
		3	11	71	9	73
		4	16	79	14	81
	20	0	0	48	0	50
		1	2	58	1	60
		2	6	67	4	69
		3	10	76	8	78
		4	14	85	12	87
	25	0	0	57	0	59
		1	2	67	1	69
		2	5	77	3	79
		3	8	86	6	88
		4	12	95	10	97
275	10	0	0	31	0	33
		1	4	40	2	42
		2	10	48	8	50
		3	16	56	14	58
		4	23	64	21	66
	12	0	0	36	0	38
		1	3	45	2	47
		2	8	54	6	56
		3	14	62	12	64
		4	20	70	18	72
	15	0	0	41	0	43
		1	3	50	2	52
		2	7	59	5	61
		3	12	67	10	69
		4	17	76	15	78
	17	0	0	45	0	47
		1	2	55	1	57
		2	7	64	5	66
		3	11	73	9	75
		4	16	81	14	83
	20	0	0	50	0	52
		1	2	60	1	62
		2	6	69	4	71
		3	10	78	8	80
		4	14	87	12	89
	25	0	0	59	0	61
		1	2	69	1	71
		2	5	79	3	81
		3	8	88	6	90
		4	12	97	10	99
300	8	0	0	26	0	28
		1	4	34	2	36
		2	10	42	8	44
		3	17	49	15	51
		4	24	56	22	58
	10	0	0	30	0	32
		1	3	39	2	41
		2	9	47	7	49
		3	15	55	13	57
		4	21	63	19	65
	12	0	0	34	0	36
		1	3	43	2	45
		2	8	52	6	54
		3	13	60	11	62
		4	19	68	17	70
	15	0	0	39	0	41
		1	3	48	2	50
		2	7	57	5	59
		3	12	65	10	67
		4	17	74	15	76
	17	0	0	43	0	45
		1	2	53	1	55
		2	7	62	5	64
		3	11	71	9	73
		4	16	79	14	81
	20	0	0	48	0	50
		1	2	58	1	60
		2	6	67	4	69
		3	10	76	8	78
		4	14	85	12	87
	25	0	0	57	0	59
		1	2	67	1	69
		2	5	77	3	79
		3	8	86	6	88
		4	12	95	10	97
325	8	0	0	26	0	28
		1	4	34	2	36
		2	10	42	8	44
		3	17	49	15	51
		4	24	56	22	58
	10	0	0	30	0	32
		1	3	39	2	41
		2	9	47	7	49
		3	15	55	13	57
		4	21	63	19	65
	12	0	0	34	0	36
		1	3	43	2	45
		2	8	52	6	54
		3	13	60	11	62
		4	19	68	17	70
	15	0	0	39	0	41
		1	3	48	2	50
		2	7	57	5	59
		3	12	65	10	67
		4	17	74	15	76
	17	0	0	43	0	45
		1	2	53	1	55
		2	7	62	5	64
		3	11	71	9	73
		4	16	79	14	81
	20	0	0	48	0	50
		1	2	58	1	60
		2	6	67	4	69
		3	10	76	8	78
		4	14	85	12	87
	25	0	0	57	0	59
		1	2	67	1	69
		2	5	77	3	79
		3	8	86	6	88
		4	12	95	10	97

APPENDIX A - CONTINUED

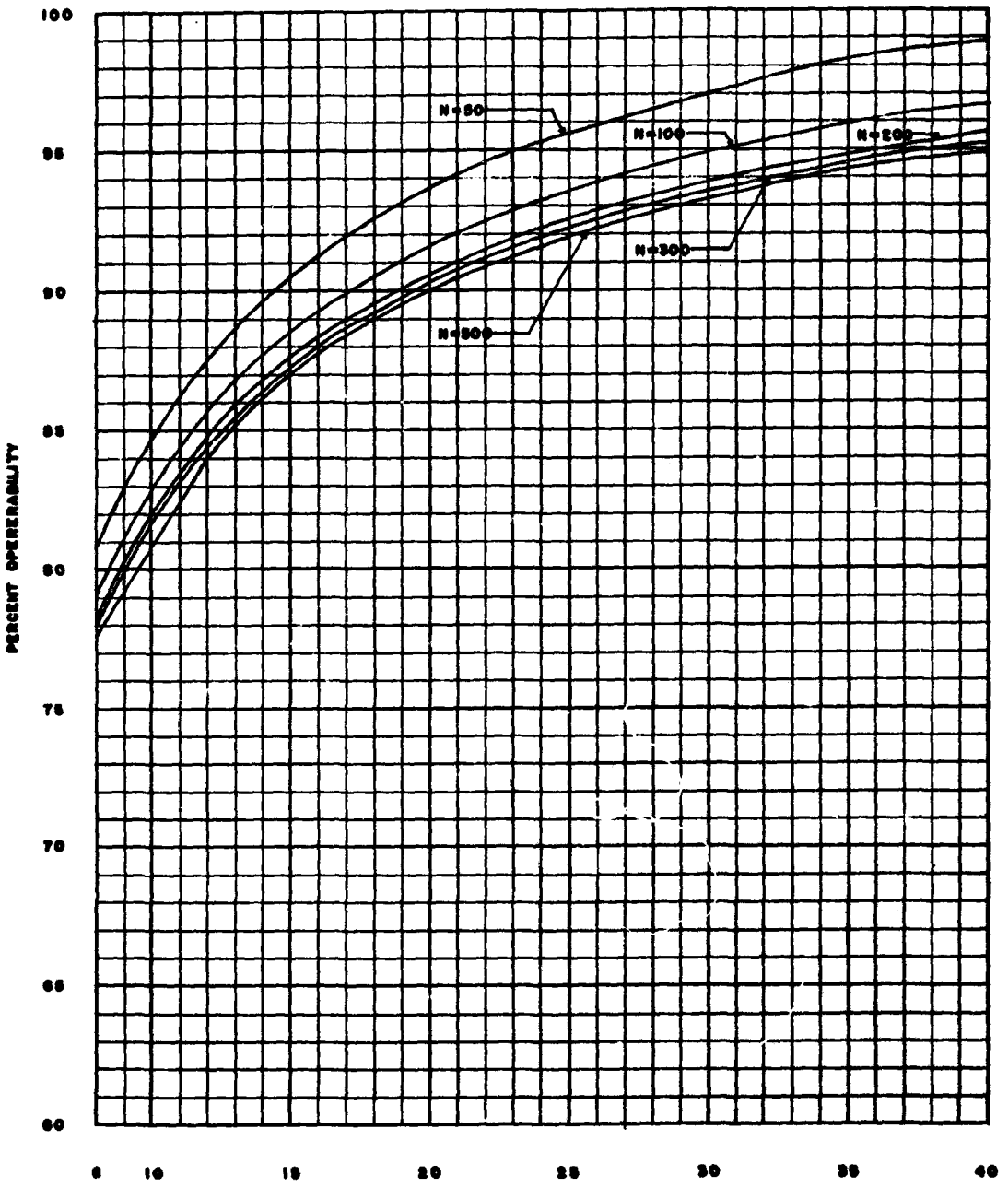
N	S	D	95% Confidence Levels M_L	95% Confidence Levels M_U	99% Confidence Levels M_L	99% Confidence Levels M_U
400	25	0	0	35	0	41
		1	2	56	1	64
		2	9	78	6	89
		3	17	98	14	110
		4	28	118	23	130
	30	0	0	27	0	35
		1	2	47	1	54
		2	7	65	6	75
		3	15	83	12	93
		4	25	100	19	110
	35	0	0	23	0	30
		1	1	40	1	46
		2	6	56	5	65
		3	12	71	10	81
		4	20	86	16	98
	40	0	0	20	0	26
		1	1	35	1	42
		2	5	49	4	57
		3	11	63	9	71
		4	17	75	14	84
600	5	0	0	100	0	126
		1	7	168	4	193
		2	34	231	25	254
		3	78	286	67	307
		4	114	336	96	353
	10	0	0	84	0	104
		1	6	141	5	164
		2	26	196	19	216
		3	56	245	45	263
		4	84	286	72	305
	15	0	0	72	0	91
		1	4	121	3	142
		2	21	168	15	187
		3	45	209	35	228
		4	70	249	58	268
	20	0	0	63	0	79
		1	4	106	2	126
		2	16	130	12	147
		3	34	175	27	195
		4	54	200	46	224
	25	0	0	58	0	67
		1	3	99	2	106
		2	14	126	10	141
		3	29	156	23	173
		4	47	187	39	204
	30	0	0	48	0	56
		1	3	78	2	90
		2	12	100	9	122
		3	25	136	20	151
		4	39	162	32	177
	35	0	0	37	0	47
		1	3	65	1	76
		2	9	87	7	100
		3	19	110	15	122
		4	31	132	25	146
	40	0	0	31	0	40
		1	2	53	1	63
		2	8	76	6	88
		3	16	96	13	108
		4	26	112	21	126
	45	0	0	26	0	34
		1	2	46	1	56
		2	7	64	5	76
		3	14	81	11	94
		4	22	97	18	107
	50	0	0	23	0	30
		1	1	39	1	48
		2	6	56	4	68
		3	12	71	10	83
		4	19	86	16	98
800	5	0	0	112	0	140
		1	6	182	5	214
		2	30	252	25	283
		3	78	315	65	341
		4	127	374	107	398

N	S	D	95% Confidence Levels M_L	95% Confidence Levels M_U	99% Confidence Levels M_L	99% Confidence Levels M_U
800	10	0	0	93	0	118
		1	6	157	4	183
		2	29	214	21	240
		3	66	270	48	293
		4	96	318	80	339
	15	0	0	80	0	101
		1	5	135	3	156
		2	23	186	17	209
		3	48	233	38	255
		4	77	276	64	297
	20	0	0	66	0	84
		1	4	111	2	131
		2	18	154	13	174
		3	37	193	30	214
		4	60	231	50	251
	25	0	0	55	0	75
		1	3	99	2	118
		2	14	139	11	156
		3	32	174	26	193
		4	53	208	43	226
	30	0	0	50	0	65
		1	3	86	2	102
		2	13	119	10	137
		3	27	151	22	168
		4	45	180	36	198
	35	0	0	41	0	53
		1	2	70	1	83
		2	10	97	8	112
		3	21	123	17	138
		4	34	147	28	162
	40	0	0	34	0	44
		1	2	59	1	70
		2	9	82	6	94
		3	16	104	14	117
		4	25	125	23	138
	45	0	0	29	0	38
		1	2	51	1	60
		2	8	71	6	82
		3	15	90	12	101
		4	24	108	20	120
	50	0	0	26	0	33
		1	2	45	1	53
		2	7	63	6	72
		3	13	79	11	90
		4	21	95	17	105
700	5	0	0	157	0	197
		1	11	262	6	302
		2	52	369	39	396
		3	109	466	88	478
		4	177	523	149	508
	10	0	0	131	0	165
		1	9	219	5	256
		2	46	303	29	337
		3	93	378	66	410
		4	154	446	112	476
	15	0	0	112	0	143
		1	7	189	4	222
		2	32	261	23	295
		3	67	327	55	356
		4	106	388	90	418
	20	0	0	96	0	119
		1	5	155	3	186
		2	23	217	18	248
		3	52	272	41	300
		4	83	324	69	348
	25	0	0	83	0	104
		1	5	135	3	166
		2	18	188	14	221
		3	40	240	32	271
		4	72	291	60	316
	30	0	0	71	0	91
		1	4	121	3	149
		2	11	168	12	195
		3	30	215	25	239
		4	53	259	45	278

N	S	D	95% Confidence Levels M_L	95% Confidence Levels M_U	99% Confidence Levels M_L	99% Confidence Levels M_U
700	25	0	0	50	0	74
		1	3	99	2	118
		2	14	137	10	157
		3	30	174	23	196
		4	47	207	39	229
	30	0	0	49	0	63
		1	3	84	2	100
		2	12	116	9	133
		3	25	147	19	164
		4	39	176	32	196
	35	0	0	43	0	54
		1	2	71	1	85
		2	10	100	7	118
		3	21	127	17	145
		4	33	152	27	168
	40	0	0	37	0	47
		1	2	63	1	75
		2	9	88	7	103
		3	18	111	15	126
		4	29	134	24	149
1000	5	0	0	225	0	282
		1	14	378	9	488
		2	75	514	56	565
		3	156	636	125	686
		4	253	748	213	797
	10	0	0	188	0	237
		1	12	315	7	366
		2	57	433	41	488
		3	110	540	90	586
		4	171	638	140	681
	15	0	0	161	0	204
		1	10	271	5	317
		2	44	374	33	419
		3	95	468	75	513
		4	153	555	127	597
	20	0	0	133	0	169
		1	7	226	4	264
		2	35	310	25	350
		3	74	390	58	430
		4	118	463	98	503
	25	0	0	119	0	152
		1	6	201	4	238
		2	31	279	22	316
		3	64	350	50	387
		4	103	417	83	455
	30	0	0	102	0	132
		1	5	173	3	204
		2	24	241	18	276
		3	54	304	42	338
		4	84	363	70	397
	35	0	0	83	0	107
		1	4	148	2	168
		2	18	197	14	225
		3	41	248	31	278
		4	67	297	55	328
	40	0	0	78	0	98
		1	4	130	2	163
		2	17	169	12	198
		3	35	221	27	254
		4	56	282	45	298
	45	0	0	68	0	78
		1	3	103	2	125
		2	14	155	10	188
		3	29	208	23	235
		4	47	258	38	288
	50	0	0	59	0	78
		1	3	91	2	105
		2	12	134	9	161
		3	24	181	19	205
		4	41	195	33	224

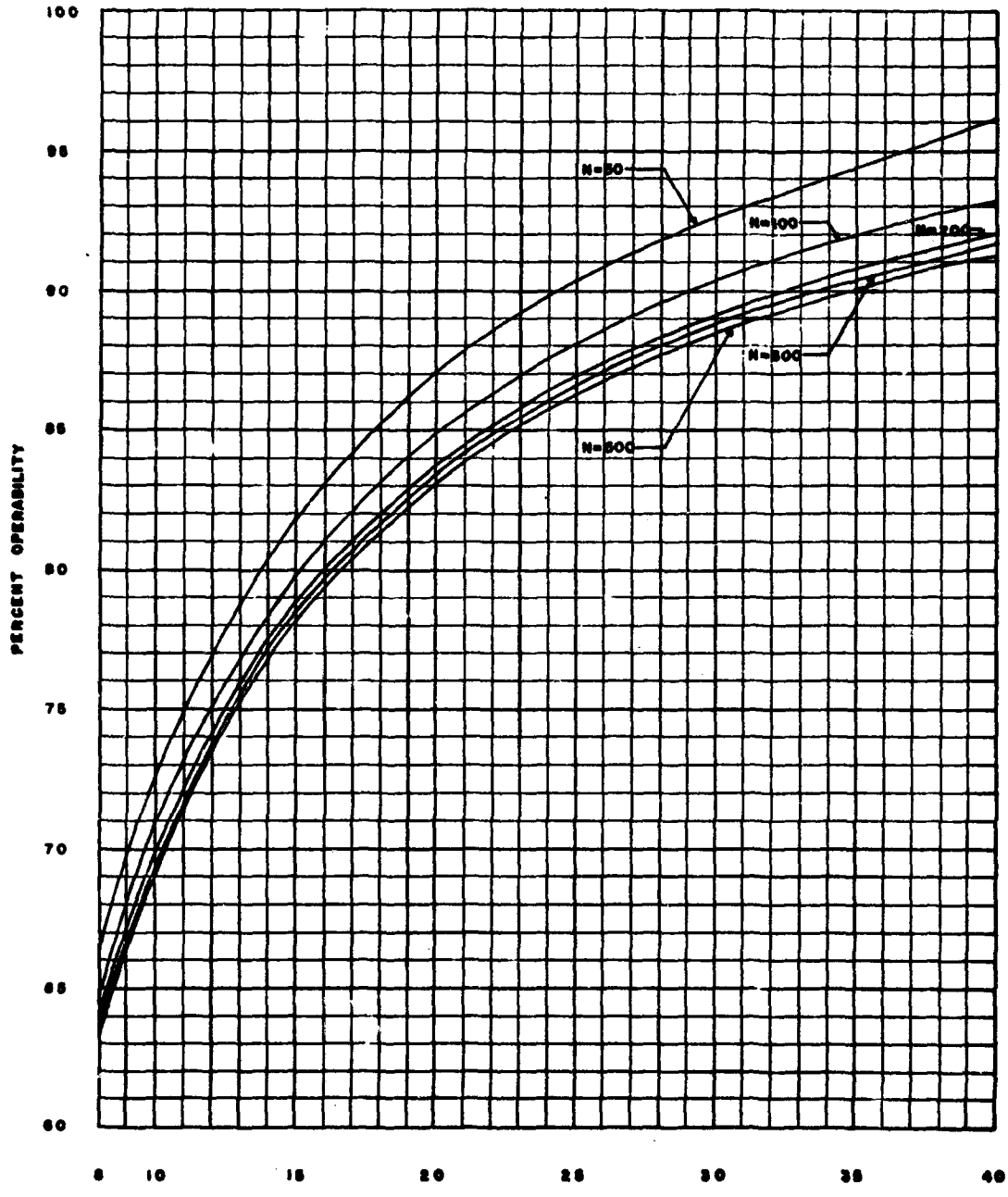
NOTE: WHERE CONFIDENCE LEVELS DO NOT FALL ON 95% OR 99%, THE NEXT NEAREST LEVELS ARE SELECTED.

APPENDIX B GRAPH I



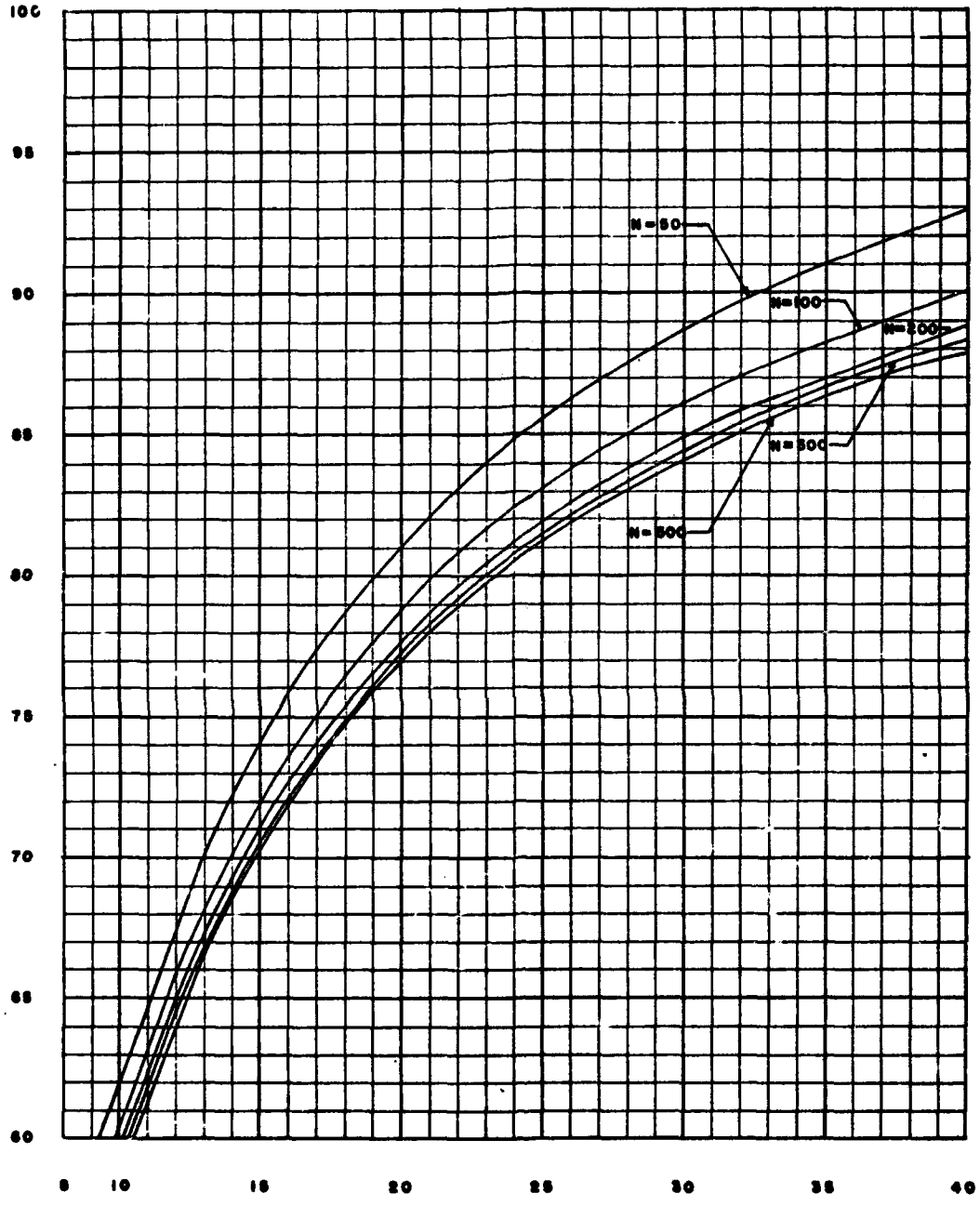
PERCENT OPERABILITY VERSUS FIXED SAMPLE SIZE
.90 CONFIDENCE LEVEL FOR NO DEFECTIVE

APPENDIX B GRAPH 2



PERCENT OPERABILITY VERSUS FIXED SAMPLE SIZE
.80 CONFIDENCE LEVEL FOR ONE DEFECTIVE

APPENDIX B GRAPH 3



PERCENT OPERABILITY VERSUS FIXED SAMPLE SIZE
.90 CONFIDENCE LEVEL FOR TWO DEFECTIVES

APPENDIX C: SAMPLE OF COMPUTER PRINT-OUT OF PROBABILITIES.

HYPERGEOMETRIC SERIES

S	N	D	R	P	SUM	COMP	INTERVAL				LEFT SUM	RIGHT SUM
4	50	0	0	0.9090909091	0.9090909091	0.0000						
4	50	0	1	0.0919999999	1.0919999999	0.1882	0	0.0000	1	0.0200	0.0000000000	0.9199999999
4	50	0	2	0.0440079991	2.7660079591	0.2710	0	0.0000	2	0.0400	0.0000000000	1.7660079591
4	50	0	3	0.7744077959	3.5393877550	0.3449	0	0.0000	3	0.0600	0.0000000000	2.5393877551
4	50	0	4	0.7005757787	4.2479635258	0.4184	0	0.0000	4	0.0800	0.0000000000	3.2479635258
4	50	0	5	0.6440604863	4.8940240121	0.4798	0	0.0000	5	0.1000	0.0000000000	3.8940240121
4	50	0	6	0.5804528875	5.4803768996	0.5376	0	0.0000	6	0.1200	0.0000000000	4.4803768996
4	50	0	7	0.5250862613	6.0202431610	0.5902	0	0.0000	7	0.1400	0.0000000000	5.0202431610
4	50	0	8	0.4860182370	6.5062613981	0.6378	0	0.0000	8	0.1600	0.0000000000	5.5062613981
4	50	0	9	0.4397307859	6.9459921841	0.6809	0	0.0000	9	0.1800	0.0000000000	5.9459921841
4	50	0	10	0.3968302214	7.3428224055	0.7198	0	0.0000	10	0.2000	0.0000000000	6.3428224055
4	50	0	11	0.3571471993	7.6999696048	0.7548	0	0.0000	11	0.2200	0.0000000000	6.6999696048
4	50	0	12	0.3205167173	8.0204863221	0.7863	0	0.0000	12	0.2400	0.0000000000	7.0204863221
4	50	0	13	0.2867781155	8.3072644376	0.8144	0	0.0000	13	0.2600	0.0000000000	7.3072644376
4	50	0	14	0.2557750750	8.5630395136	0.8395	0	0.0000	14	0.2800	0.0000000000	7.5630395136
4	50	0	15	0.2273556231	8.7903951367	0.8618	0	0.0000	15	0.3000	0.0000000000	7.7903951367
4	50	0	16	0.2013721233	8.9917672600	0.8815	0	0.0000	16	0.3200	0.0000000000	7.9917672600
4	50	0	17	0.1776812852	9.1694485453	0.8989	0	0.0000	17	0.3400	0.0000000000	8.1694485453
4	50	0	18	0.1561441907	9.3255927051	0.9142	0	0.0000	18	0.3600	0.0000000000	8.3255927051
4	50	0	19	0.1366261398	9.4622188449	0.9276	0	0.0000	19	0.3800	0.0000000000	8.4622188449
4	50	0	20	0.1189969684	9.5812158854	0.9393	0	0.0000	20	0.4000	0.0000000000	8.5812158854
4	50	0	21	0.1031306990	9.6843465845	0.9494	0	0.0000	21	0.4200	0.0000000000	8.6843465845
4	50	0	22	0.0889057750	9.7732522796	0.9581	0	0.0000	22	0.4400	0.0000000000	8.7732522796
4	50	0	23	0.0762049500	9.8494572296	0.9656	0	0.0000	23	0.4600	0.0000000000	8.8494572296
4	50	0	24	0.0649153275	9.9143725575	0.9719	0	0.0000	24	0.4800	0.0000000000	8.9143725575
4	50	0	25	0.0549283543	9.9693009118	0.9773	0	0.0000	25	0.5000	0.0000000000	8.9693009118
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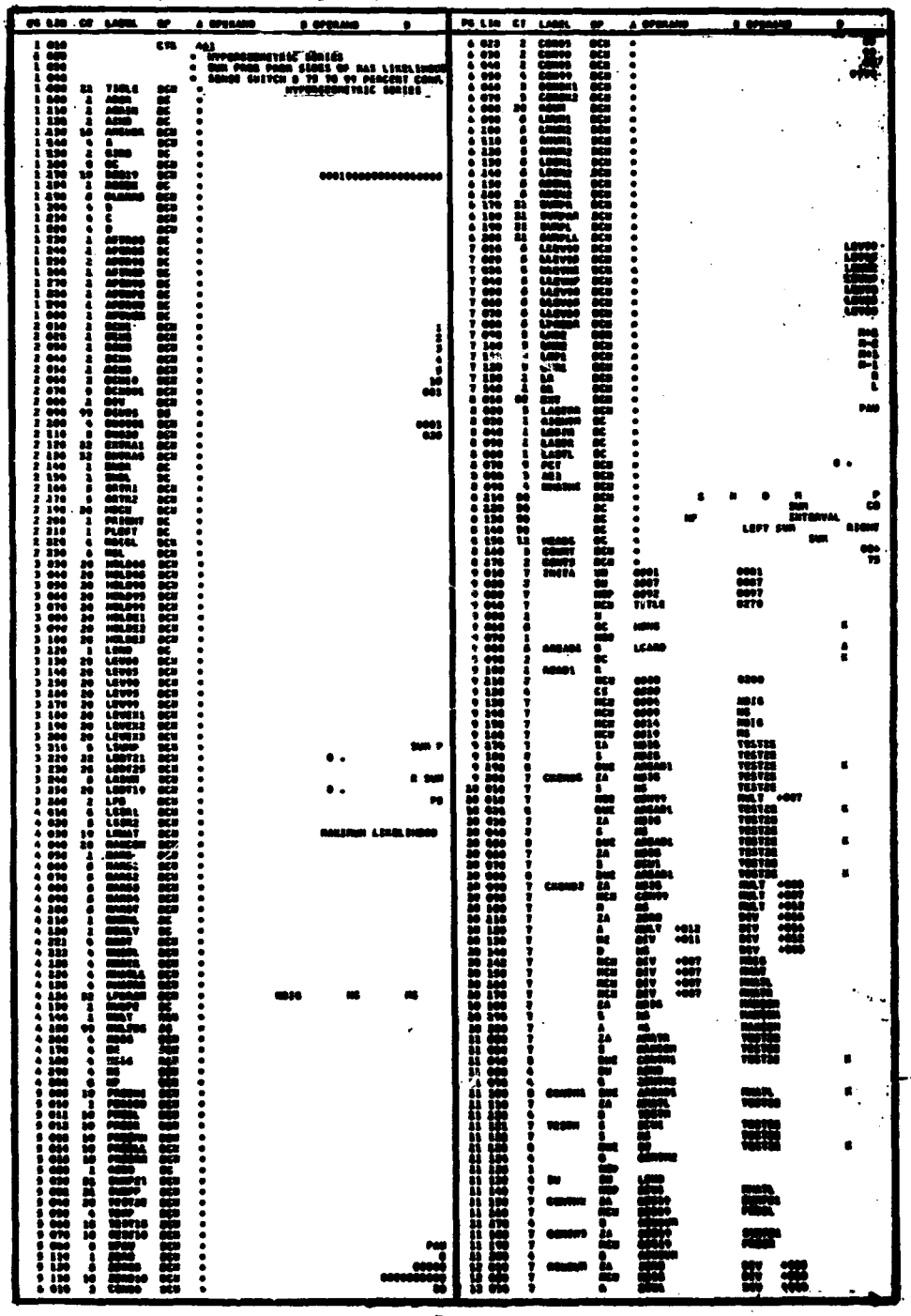
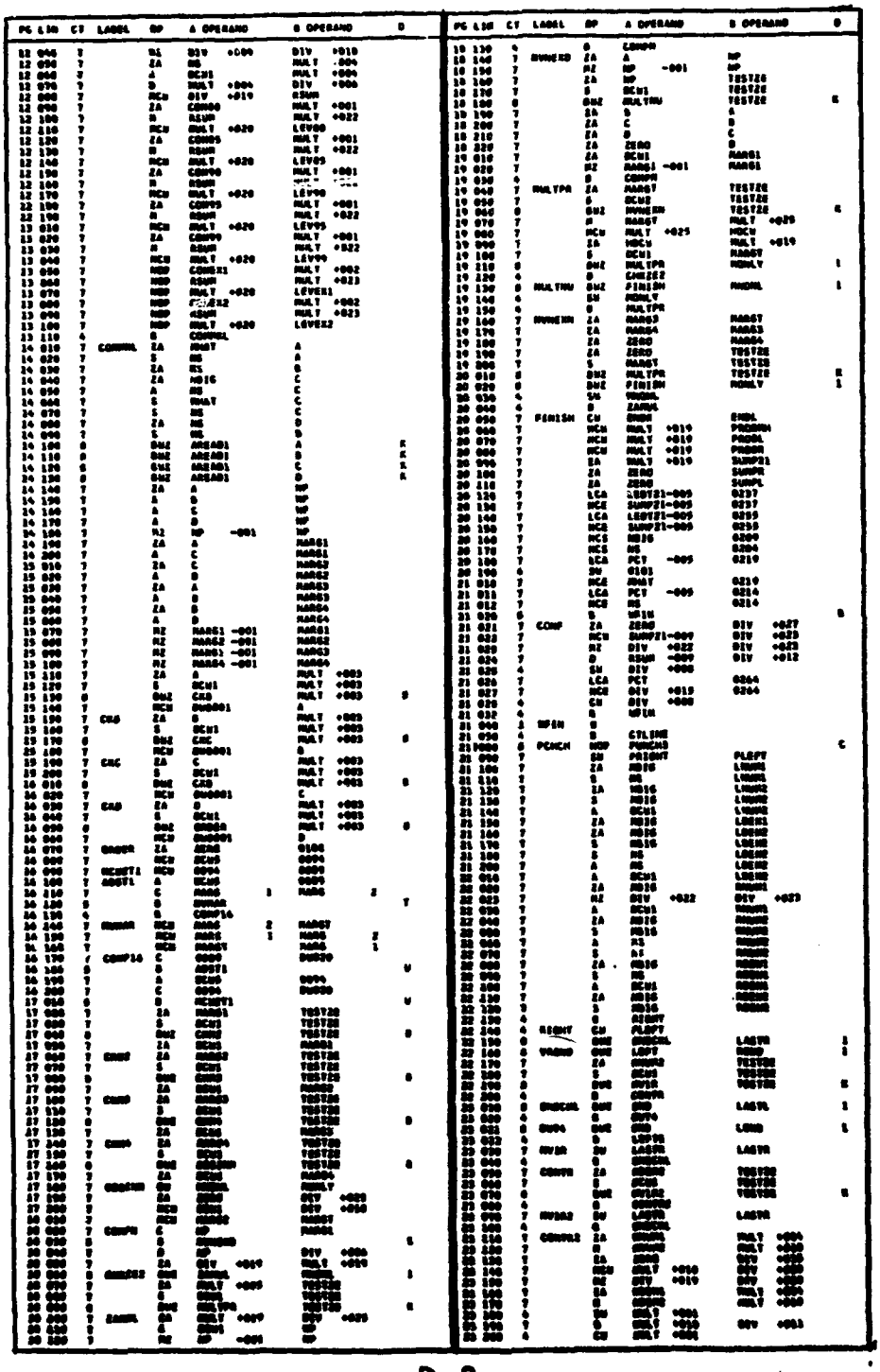
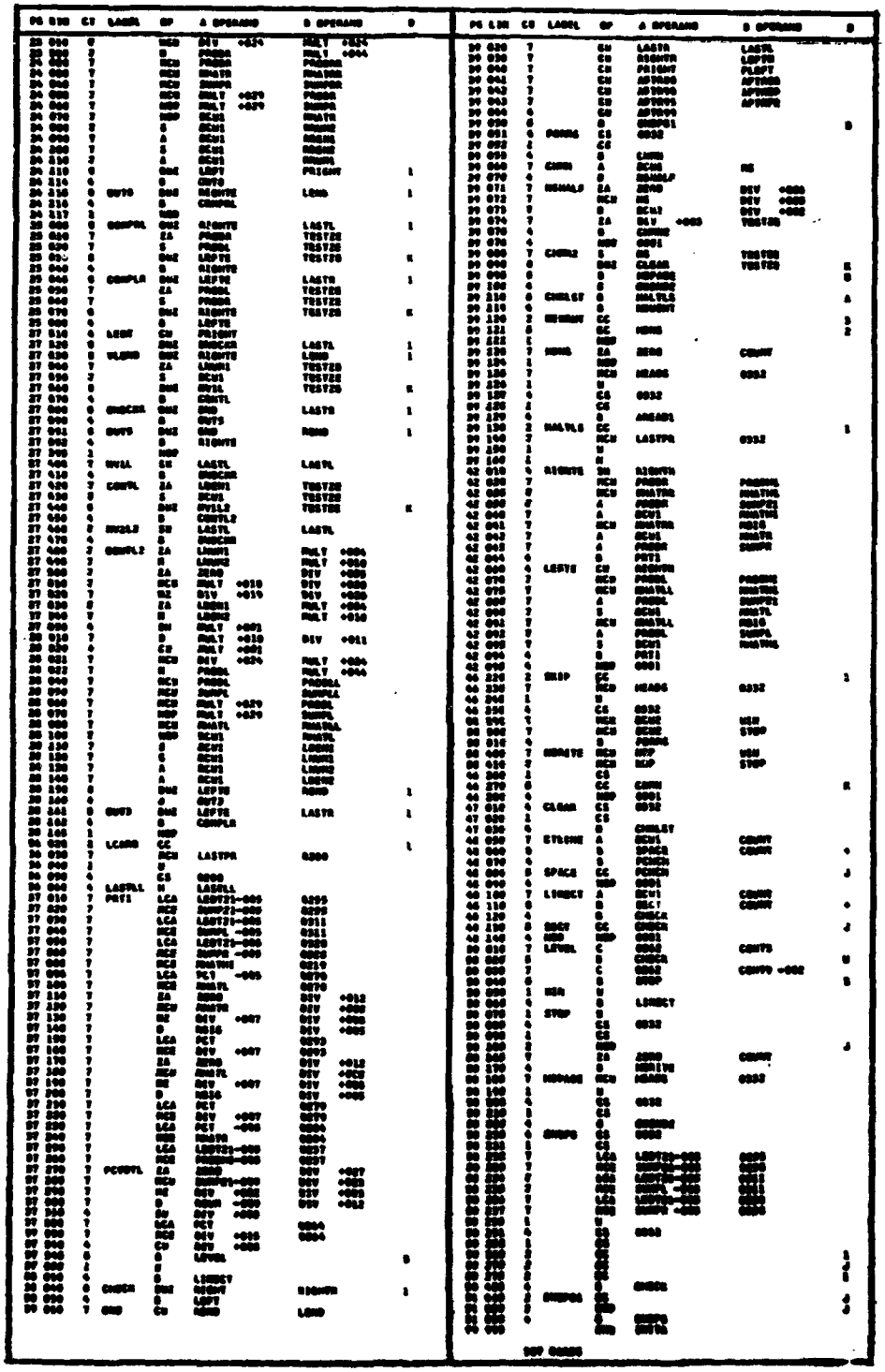


APPENDIX D: CONTINUED.



APPENDIX B CONTINUED.



U. S. NAVAL AMMUNITION DEPOT
OAHU, HAWAII

1804:sm
5220/NT
3 Jan 1963

From: Commanding Officer
To: Chief, Bureau of Naval Weapons (FQ-1)
Department of the Navy
Washington 25, D.C.

Subj: Tables of the Hypergeometric Distribution Functions

Encl: (1) Mathematical derivation of the tables
(2) Tables of point and cumulative probabilities for various sample and population size combinations

1. Enclosures (1) and (2) are forwarded for publication by the U. S. Government Printing Office. Enclosure (1) contains mathematical derivation of the hypergeometric distribution function, tables, graphs and IBM 1401 computer program. Enclosure (2) (forwarded under separate cover) contains two copies of the tabulations of the hypergeometric probabilities.

2. The tables, which include point and cumulative probabilities are designed primarily for use by personnel familiar with statistics to estimate stockpile quality level.

3. Copy addressees are advised that copies of the tabular presentation, enclosure (2), which are quite voluminous cannot be made available by this command. It is presumed that they will be generally available through the Bureau of Naval Weapons or the Government Printing Office if the Bureau of Naval Weapons decides to have them published.

H. H. MEEKER, JR.
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